Analytical Relationship Between Feedback Loop Properties and Load Transient Response in Voltage Converters

Many applications impose strict requirements on output voltage response to rapid load changes, motivating an analytical link between frequency-domain loop design and time-domain transient behavior. Phase margin and feedback loop bandwidth are key design parameters in voltage converter control for achieving specified transient performance. Existing models are typically based on specific converter topologies or controller methods, limiting their portability. This article presents a generic loop model formulated directly in terms of loop design parameters, largely independent of specific converter implementation.

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In frequency domain small-signal output voltage excursion ΔV(s) in response to load current step ΔI is given by:

$$\Delta V(s) = -\frac{\Delta I}{s} \times \frac{Z_{OL}(s)}{1+T(s)} \qquad (1)$$

where $Z_{OL}(s)$ is the open-loop output impedance, T(s) – open loop transfer function [1].

Since inductors can't follow rapid current changes, the transient current flows primarily through output capacitance. Therefore for step load transient analysis we can assume that the open-loop output impedance is dominated by capacitance C. Then:

$$\Delta V(s) \approx -\frac{\Delta I}{s} \times \frac{1}{sC \times (1+T(s))} \qquad (2)$$

For a given T(s) one can solve (2) for the step load response by taking the inverse Laplace transform. Classical approaches based on detailed plant–compensator models yield high-order transfer functions, which make symbolic solutions unwieldy and obscure the direct influence of key design parameters. Besides this, these models are usually based on specific converter topologies or controller structures, limiting their portability. In engineering practice, it is therefore common to approximate the open-loop transfer function by a second-order form with a single pole at the origin and another high-frequency pole above crossover [1], [2]. This approximation is a cornerstone of classical control theory and is well suited for *input* steps analysis which deals with T/(1+T). However, when applied to *load* step analysis which deals with 1/(1+T) according to (2), it yields a steady-state DC voltage offset and distortion of the predicted waveform because $Z_{CL}(s)$ does not vanish as s→0.

Here, we construct a loop gain formulation that enforces zero DC output impedance while yielding a compact and analytically tractable expression for the transient response in terms of bandwidth, phase margin and output capacitance.

Bandwidth (also called crossover frequency) ω_C is defined as the frequency at which the magnitude |T(jω)| equals unity (or 0 dB in logarithmic scale). Phase margin is the difference between the argument of T(s) at the crossover frequency and –180°:

$$\varphi_M = \angle T(j\omega_C) + 180^0 \qquad (3)$$

It's well-known that phase margin φ_M and consequently the transient response, are primarily determined by the local slope of T(s) magnitude at ω_C [1], [3]. To analyze the transient response we aim to derive a reverse relationship: to reconstruct T(s) which results in particular phase margin at given crossover frequency.

Construction Of Open Loop Transfer Function

In well-stabilized voltage converters characterized by *minimum-phase* behavior the loop gain T(s) displays a smooth and monotonic magnitude characteristic in the vicinity of ω_C. For such systems, the Bode gain–phase relationship, which follows from the Hilbert transform dictates a direct correspondence between the phase margin and the local slope of the gain magnitude at crossover [3], [4].

Let's consider two boundary cases:

- $T(s)=\omega_C/s$ has slope m=–1 or –20 dB/decade yielding phase margin φ_M=90° (stable condition).
- $T(s)=(\omega_C/s)^2$ has slope m=–2 or –40 dB/decade yielding phase margin φ_M=0° (oscillatory-like).

Between these two boundaries the local slope "m" varies between –1 and –2 while the resulting phase margin varies between 90° and 0°.

Therefore, in general, for a stabilized converter the T(s) near ω_C can be approximated as a fractional order function:

$$T(s) = \left(\frac{\omega_C}{s}\right)^n \qquad (4)$$

where 1<n<2.
On log-log scale magnitude of such a function has slope m=–n.

By substituting s=j×ω we find phase of the function (4):

$$\angle T(j\omega) = -n \times \frac{\pi}{2} = -n \times 90^o \qquad (5)$$

Then from (3) phase margin is:

$$\varphi_M = 180^o - n \times 90^o \qquad (6)$$

Rearranging (6) we express power "n" of function (5) in terms of phase margin:

$$n = 2 - \frac{\varphi_M}{90^o} \qquad (7)$$

Theoretically, by substituting (4) into (2) and taking inverse Laplace transformation one can derive step load transient response. However, with the fractional-order model (4) the result includes Mittag-Leffler function which masks direct influence of key design parameters.

To avoid this complexity while preserving the essential closed loop behavior, we will construct a dynamically equivalent function T(s) that behaves similar to (4) around the crossover frequency. Namely, we need T(s) that satisfies three invariants at ω_C: unity gain, prescribed phase according to (3), and a magnitude slope approximating that of (7).

Let us examine the following function:

$$T(s)=\frac{\omega_C}{s}\times\frac{\sin\varphi_M \times s+\cos\varphi_M \times \omega_C}{s} \qquad (8)$$

The first term in (8) represents m=–1 slope (-20 dB/decade) corresponding to φ_M=90°. The second one is a factor that "shapes" the slope depending on φ_M like "fractional" slope of (4).

To verify the accuracy of the model (8) we will calculate its magnitude, slope and phase at ω_C. By substituting s=j×ω_C it is easy to find that for such function $|T(j\omega_C)|$ = 1.

The local slope of its magnitude at ω_C on a log-log scale:

$$m = \frac{d(\log|T(j\omega|)}{d(\log\omega)}|_{\omega=\omega_C} = \sin{\varphi_M}^2 - 2 \qquad (9)$$

By comparing (9) to desired slope m=–n, where "n" is given by (7), one can find that the function (8) provides exact slope match at 0°, 45°, and 90°. The maximum discrepancy occurs at φ_M ≈22.5° and 67.5°, where the slope error is less than 6% (approximately 1.2 dB/decade).

The phase of the function (9) at ω_C after some algebra is given as:

$$\angle T(j\omega_c) = -180^o + \varphi_M \qquad (10)$$

Equations (9) and (10) demonstrate that the phase of the function (8) at ω_C is exactly related to φ_M (3), and the slope of its magnitude closely approximates desired slope. Therefore the function (8) can adequately approximate behavior of stabilized converters open loop transfer function near crossover frequency.

Transient Response Calculation

The step load response is obtained by substituting (8) into (2) and applying the inverse Laplace transform. After some mathematics one can find that the resulting expression exhibits three distinct regions depending on the value of $(\cos(\varphi_M)\text{-}\sin(\varphi_M)^2/4)$. When $\cos(\varphi_M)\text{-}\sin(\varphi_M)^2/4=0$, which occurs at φ_M≈76.345°, the response is critically damped. The response is underdamped for φ_M<76.345° and overdamped for φ_M>76.345°.

Remarkably, the critical phase margin, obtained here independently of converter topology within the adopted model assumptions, coincides with the optimal value 76° reported in [2] for a buck converter.

The main practical interest presents underdamped region. In this region by applying the inverse Laplace transform one can find the transient recovery of the output voltage V(t):

$$V(t) = -\frac{\Delta I}{C\times\omega_D}\times e^{-\sigma t}\times\sin(\omega_D t) \qquad (11)$$

where $\omega_D = \omega_C \times \sqrt{(\cos\varphi_M - \frac{1}{4}\sin{\varphi_M}^2)}$ – natural frequency,

$\sigma = \frac{\omega_C \times \sin\varphi_M}{2}$ – decay constant, φ_M<76.345°.

Figure 1 provides examples of transient response calculated from (11) for load step ΔI=10 amps, output capacitance C=470 μF, and crossover frequency F_C=10 kHz (which is $\omega_C=2\pi F_C$≈62832 radians).

The curves of Figure 1 show that the phase margin mostly affects the recovery shape and does not affect much peak deviation. The peak deviation depends primarily on the bandwidth and the capacitance which may be selected based on (11).

To validate the proposed bandwidth- and phase-margin–based model we simulated the transient response of 3.3V forward converter by using LTspice average model [5]. The component values in the compensation circuit were automatically adjusted for various phase margins at fixed crossover frequency. The resulting phase margins were verified with AC analysis. The simulation was done under the same conditions of load step, bandwidth and output capacitance as used for Figure 1.

The simulation results confirm that the general dependence of the recovery waveform on phase margin follows the analytical derivation. One can see from the simulation plots that phase margin close to 76° yields practically aperiodic recovery without overshoot as predicted above.

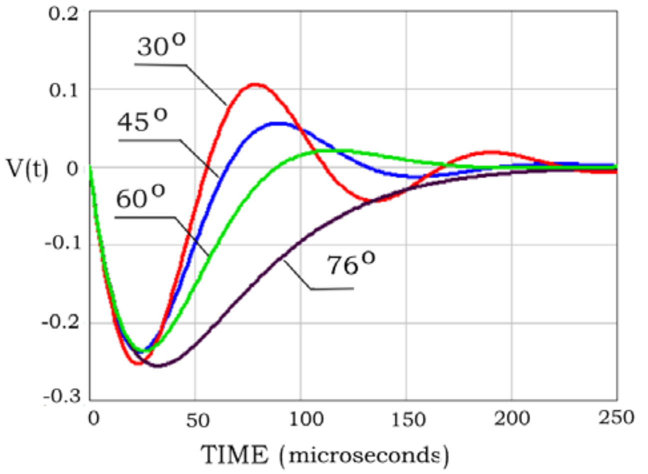

Figure 1: Step load transient response (in volts) at various phase margins

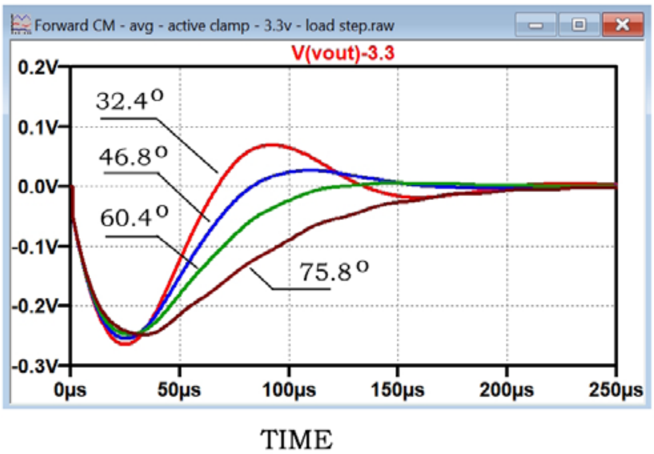

Figure 2: Simulated step load voltage transient response at various measured phase margins

Equation (11) also allows estimation of the desired characteristics of the transient voltage such as settling time and peak overshoot, as functions of three properties: ω_C, φ_M, and C.

We define settling time as the time from the initial onset of the load transient to the time when V(t) returns to a specified fraction "ε" of the nominal steady-state value Vo.

$$|V(t)| \le \varepsilon \times Vo \;\; \exists\, t > t_{PK} \tag{12}$$

To find the settling time we will first identify time to peak, as the time where the first derivative of V(t) is zero. By setting dV(t)/dt=0 and solving for "t" after some manipulations we find time to peak as:

$$t_{PK} = \frac{1}{\omega_D} \times \operatorname{atan}\left(\frac{\omega_D}{\sigma}\right) \tag{13}$$

Then from (11) the absolute values of the peak deviation Vpk is:

$$V_{PK} = \frac{\Delta I}{C \times \omega_D} \times e^{-\sigma t_{PK}} \times \sin(\omega_D t_{PK}) \tag{14}$$

The decay time can be estimated as the time from peak to the time when the envelope of V(t) returns to a specified voltage band:

$$td = \frac{1}{\sigma} \ln\left(\frac{V_{PK}}{\varepsilon \times Vo}\right) \tag{15}$$

Then total settling time can be found as:

$$t_S = t_{PK} + td \tag{16}$$

By combining (13) - (16) after some mathematical manipulations we obtain the settling time:

$$t_S = \frac{1}{\sigma} \ln \frac{\Delta I}{C \times \omega_C \sqrt{\cos \varphi_M} \times \varepsilon Vo} \tag{17}$$

Figure 3 provides examples of settling time for Vo=3.3 volt and ε=0.01 (1%) for various crossover frequencies under the same load step and output capacitance as for Figure 1.

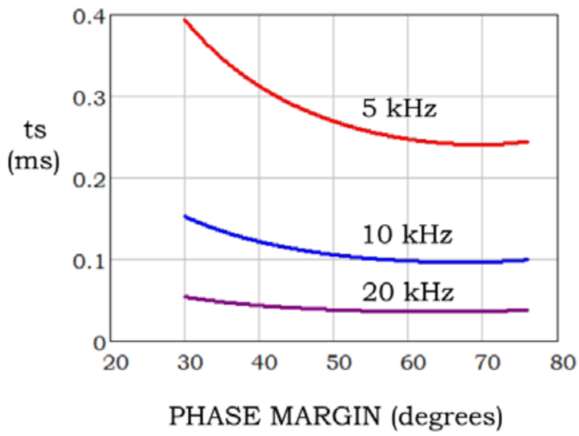

Figure 3: Settling time (in milliseconds) at various crossover frequencies

To estimate peak overshoot (i.e. the peak voltage after it reverses polarity) we note that the time span between the peak deviation t_{PK} and the subsequent polarity-reversed peak is exactly half of the period of the natural frequency ω_D:

$$t_{OS} - t_{PK} = \frac{\pi}{\omega_D} \tag{18}$$

By combining (11) and (18) we find overshoot magnitude:

$$V_{OS} = V_{PK} \times e^{-\sigma\pi/\omega_D} \tag{19}$$

Figure 4 provides examples of the overshoot for various crossover frequencies under the same load step and output capacitance as for Figure 1.

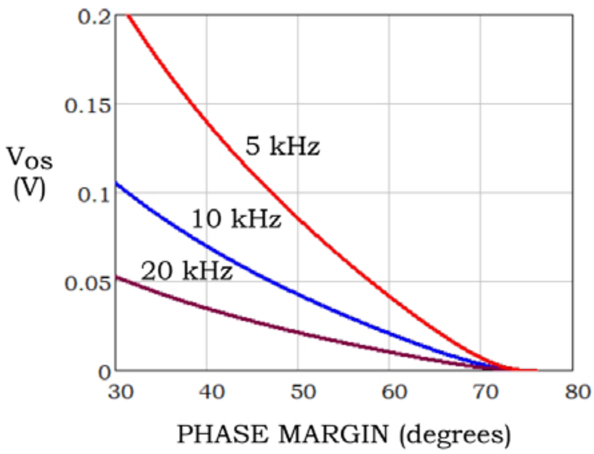

Figure 4: Overshoot (in volts) at various crossover frequencies

The preceding analysis is based on the response to step load where transition mathematically occurs instantly. In practice, of course, load transitions occur over some non-zero time. With a finite slew rate di/dt assuming load change is still much faster than the response time, the actual time to peak is given as [6]:

$$t'_{PK} = t_{PK} - \frac{\Delta I}{di/dt} \tag{20}$$

As the result, the peak deviation, settling time and overshoot value are reduced accordingly.

Owing to the simplifying assumptions used in the analysis, the obtained expressions are intended for quick qualitative estimation of transient behavior. Consequently, the derived expressions should be interpreted as first-order analytical approximations intended to capture the transient behavior and provide design insight, rather than exact quantitative predictions.

Conclusion

The proposed approach avoids detailed circuit-level derivations by treating the loop as a behavioral system described by its phase margin, crossover frequency (bandwidth), and output capacitance. A generic, topology-independent representation of the loop gain T(s) in the vicinity of the crossover frequency enables a simple direct mapping between frequency-domain design parameters and time-domain transient behavior. It was found particularly that phase margin close to 76° yields aperiodic recovery.

The results provide a practical means for establishing design targets that meet output voltage transient requirements, without need of the full circuit-level analysis.

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